

мые условия существования решения рассматриваемой задачи и предложен алгоритм нахождения их решения.

**Ключевые слова:** краевые задачи, система дифференциальных уравнений, конформная производная, конформный интеграл, метод параметризации.

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**ON ONE PROPERTY OF A SOLUTION OF  
A THIRD ORDER PSEUDOPARABOLIC EQUATION**

*Abstract.* Sufficient conditions for the boundedness of the highest derivative of a third-order partial differential equation are established and a coercive estimate is obtained in the norm of the space  $C_*(\bar{\Omega}, R)$ .

*Key words:* pseudoparabolic equation, diagonal dominance, coercive estimate.

1. **Introduction.** Let  $\bar{\Omega} = [0, \omega] \times (-\infty, +\infty)$ . Consider the third-order pseudoparabolic equation

$$u_{xxt} = a_0(x, t)u_{xxt} + a_1(x, t)u_x + a_2(x, t)u_t + a_3(x, t)u + f(x, t) \quad (1)$$

where the functions  $a_i(x, t)$  ( $i = \overline{0,3}$ ),  $f(x, t)$  are assumed to be continuous and, generally speaking, unbounded on  $\bar{\Omega}$ .

At present, studies of local and nonlocal boundary value problems for equation (1) are very actively studied and arouse great practical and theoretical interest due to the fact that applied problems of physics, mechanics, and biology are reduced to such equations.

Local and nonlocal boundary value problems for pseudoparabolic equations of the third order are investigated in the works of M.Kh. Shkhanukov. [1-4]. A.M.Nakhushev [5-8] pointed out examples of the practical application of the results of the study of boundary value problems for the equations under study in the study of moisture transfer processes in porous media and in problems of mathematical biology.

Nonlocal problems for pseudoparabolic equations of the third order were also considered in [9–11], where A.A. Samarskii investigated them by the method of passing from a problem with a nonclassical boundary condition to a problem with a classical condition, but for a nonclassical equation there is the so-called loaded equation [12].

In this paper, in contrast to the studies above, we consider the case of equation (1) with continuous and generally speaking unbounded coefficients given in an infinite domain. New problems arising due to the unboundedness of the coefficients of differential equations in the 1970s gave impetus to the creation of the theory of separability in the works of the English mathematicians Everitt and Geertz [13,14]. The main task of the separability theory is to obtain nonlocal weighted estimates of the solution and its derivatives in the corresponding function spaces, called coercive estimates of the solution. Separability conditions for an ordinary differential operator of the second order in the space of continuous and bounded functions on the entire axis by the parametrization method were established in the works of D.S. Dzhumabaev and R.A. Medetbekova [15], D.S. Dzhumabaev [16] and M.M. Medetbekov [17]. The works of M. Otelbaev and A. Birgebaev [18], T.T. Amanova [19], M.B. Muratbekov and Zh.Zh. Aitkozha [20] are devoted to the establishment of coercive estimates for the solution of a singular differential equation of the third order. Multidimensional boundary value problems for them were systematically studied in the works of T.D. Dzhuraev [21] and others.

**2. Ancillary results.** We denote by  $C_*(\overline{\Omega}, R)$  the space of bounded functions continuous with respect to  $t \in R$  uniformly with respect to  $t \in R$  and continuous with respect to  $x \in [0, \omega]$ . Let  $\|V(x, \cdot)\|_1 = \sup_{t \in R} \|V(x, t)\|$ , where  $\|V(x, t)\| = \max_{i=1, n} |V_i(x, t)|$ .

The properties of the solution  $u(x, t)$  to equation (1), satisfy the conditions

$$u(0, t) = \psi(t), \quad u(x, t), u_x(x, t), u_t(x, t), u_{xt}(x, t) \in C_*(\overline{\Omega}, R) \quad (2)$$

We put  $P_{\alpha, \beta}(x, t) = \frac{\alpha(x, t)}{\sqrt{\beta(x, t)}}$ ,  $\theta(x, t) = \frac{1}{d} \int_t^{t+d} a_1(x, \tau) d\tau$ .

The following result was established in [22].

**Theorem 1.** Let the functions  $a_i(x, t)$  ( $i = \overline{0, 3}$ ) of equation (1) be continuous on  $\overline{\Omega}$ ,  $\psi, \dot{\psi}, \ddot{\psi}$  continuous and bounded on  $R$  and the following conditions are satisfied:

a)  $a_1(x, t) \geq \gamma > 0$ ,  $\gamma$  is constant;

b)  $\frac{a_1(x, t)}{a_1(x, \bar{t})} \leq c$  at  $t, \bar{t} \in R: |t - \bar{t}| < d$ ,  $c, d$  is constant;

c) for each  $\varepsilon > 0$  there is  $\delta > 0$ , a number such that the inequality  $\left| \frac{a_1(x', t) - a_1(x'', t)}{a_1(x'', t)} \right| < \varepsilon$  holds for

all  $t$  from  $R$  and  $x', x'' \in [0, \omega]: |x' - x''| < \delta$ .

d)  $P_{a_0, a_1}(x, t) \leq K, P_{a_2, a_1}(x, t), P_{a_3, a_1}(x, t), P_{f, a_1}(x, t) \in C_*(\overline{\Omega}, R)$ .

Then there is a unique solution  $u(x, t)$  to problems (1) and (2) and the following estimate is valid:

$$\max \left\{ \|u(x, \cdot)\|_1, \|u_x(x, \cdot)\|_1, \|u_t(x, \cdot)\|_1, \|u_{xt}(x, \cdot)\|_1 \right\} \leq \tilde{C}$$

where  $\tilde{C} - const$ , depending only on the norms of functions  $f, \psi$ , of constants  $\gamma, K, c, d, \varepsilon$ .

**3. Main result.** When the coefficients of equation (1) are not bounded, this theorem cannot guarantee the boundedness of the derivative  $u_{xt}(x, t)$ . The purpose of this work is to find conditions for belonging  $u_{xt}(x, t)$  to a space  $C_*(\overline{\Omega}, R)$  and obtain an estimate for  $\|u_{xt}(x, \cdot)\|_1$ .

**Theorem 2.**

Let the assumptions of Theorem 1 and be satisfied  $f(x,t), \sqrt{\theta(x,t)}\psi(t), \sqrt{\theta(x,t)}\dot{\psi}(t) \in C_*(\bar{\Omega}, R^2)$ .

Then there is a unique solution  $u(x,t)$  to problems (1) and (2), moreover  $u_{xt} \in C_*(\bar{\Omega}, R)$  and the estimate

$$\|u_{xt}\|_1 + \|a_0 u_{xt}\|_1 + \|a_1 u_x\|_1 + \|a_2 u_t\|_1 + \|a_3 u\|_1 \leq C. \tag{3}$$

Here  $C$  depends on the norms of functions  $f, \psi$ , of constants  $\gamma, K, c, d, \varepsilon$ .

**Proof.** We introduce new functions  $u_1(x,t)$  and  $u_2(x,t)$

$$u_1(x,t) = u(x,t), \quad u_2(x,t) = u_t(x,t) \tag{3}$$

and put  $U(x,t) = (u_1(x,t), u_2(x,t))$ ,  $V(x,t) = U_x(x,t)$ .

Then problem (1), (2) is reduced to the following problem

$$V_t(x,t) = A(x,t)V + C(x,t)U(x,t) + F(x,t), \quad (x,t) \in \bar{\Omega}, \quad V(x,t) \in C_*(\bar{\Omega}, R^2), \tag{4}$$

$$U(x,t) = \Psi(t) + \int_0^x V(\xi,t) d\xi, \quad (x,t) \in \bar{\Omega}, \tag{5}$$

where

$$A(x,t) = \begin{pmatrix} 0 & 1 \\ a_1(x,t) & a_0(x,t) \end{pmatrix}, \quad C(x,t) = \begin{pmatrix} 0 & 0 \\ a_3(x,t) & a_2(x,t) \end{pmatrix},$$

$$F(x,t) = \begin{pmatrix} 0 \\ f(x,t) \end{pmatrix}, \quad \Psi(t) = \begin{pmatrix} \psi(t) \\ \dot{\psi}(t) \end{pmatrix}.$$

If the vector function  $U(x,t)$  is known, then solving problem (4) we find  $V(x,t)$ . If  $V(x,t)$  known, then from (5) we find  $U(x,t)$ . Then  $u(x,t) = u_1(x,t)$  is a solution to problem (1), (2).

To find a solution to problem (4), (5), we use the method of successive approximations.

We take the function  $U(x,t)$  as the zero approximation  $\Psi(t)$ , and find the function  $V^{(0)}(x,t) = (v_1^{(0)}(x,t), v_2^{(0)}(x,t))$  as a solution to the problem

$$V_t^{(0)}(x,t) = A(x,t)V^{(0)} + C(x,t)\Psi(t) + F(x,t), \quad V^{(0)}(x,t) \in C_*(\bar{\Omega}, R^2). \tag{6}$$

$U^{(0)}(x,t)$  we find from the relation

$$U^{(0)}(x,t) = \Psi(t) + \int_0^x V^{(0)}(\xi,t) d\xi. \tag{7}$$

By the equalities

$$v_1^{(0)}(x,t) = \tilde{v}_1^{(0)}(x,t) + \tilde{v}_2^{(0)}(x,t), \quad v_2^{(0)}(x,t) = K\sqrt{\theta(x,t)}(\tilde{v}_1^{(0)}(x,t) - \tilde{v}_2^{(0)}(x,t)), \tag{8}$$

we introduce new unknown functions  $\tilde{v}_1^{(0)}(x,t), \tilde{v}_2^{(0)}(x,t)$ . As  $\theta(x,t) \geq \gamma > 0$ , since transformation (8) is reversible and

$$\tilde{v}_1^{(0)}(x,t) = \frac{1}{2} \left[ v_1^{(0)}(x,t) + \frac{v_2^{(0)}(x,t)}{K\sqrt{\theta(x,t)}} \right], \quad \tilde{v}_2^{(0)}(x,t) = \frac{1}{2} \left[ v_1^{(0)}(x,t) - \frac{v_2^{(0)}(x,t)}{K\sqrt{\theta(x,t)}} \right].$$

As a result, (6) is reduced  $\tilde{V}^{(0)}(x,t) = (\tilde{v}_1^{(0)}(x,t), \tilde{v}_2^{(0)}(x,t))$  to the problem

$$\tilde{V}_t^{(0)}(x,t) = \tilde{A}(x,t)\tilde{V}^{(0)} + \tilde{C}(x,t)\Psi(t) + \tilde{F}(x,t), \quad \tilde{V}^{(0)}(x,t) \in C_*(\bar{\Omega}, R^2). \tag{9}$$

Here  $\tilde{A}(x,t) = (\tilde{a}_{ij}(x,t))_{i,j=1}^2$ ,  $\tilde{C}(x,t) = (\tilde{c}_{i,j}(x,t))_{i,j=1}^2$ ,

$$\tilde{a}_{11}(x,t) = \frac{1}{2} \left[ K\sqrt{\theta(x,t)} + \frac{a_1(x,t)}{K\sqrt{\theta(x,t)}} + a_0(x,t) - \frac{\theta_t(x,t)}{2\theta(x,t)} \right],$$

$$\tilde{a}_{12}(x,t) = -\frac{1}{2} \left[ K\sqrt{\theta(x,t)} + \frac{a_1(x,t)}{K\sqrt{\theta(x,t)}} - a_0(x,t) + \frac{\theta_t(x,t)}{2\theta(x,t)} \right],$$

$$\begin{aligned} \tilde{a}_{21}(x,t) &= \frac{1}{2} \left[ K\sqrt{\theta(x,t)} - \frac{a_1(x,t)}{K\sqrt{\theta(x,t)}} - a_0(x,t) + \frac{\theta_t(x,t)}{2\theta(x,t)} \right], \\ \tilde{a}_{22}(x,t) &= -\frac{1}{2} \left[ K\sqrt{\theta(x,t)} - \frac{a_1(x,t)}{K\sqrt{\theta(x,t)}} + a_0(x,t) - \frac{\theta_t(x,t)}{2\theta(x,t)} \right], \\ \tilde{c}_{1j}(x,t) &= \frac{1}{2} \left[ \frac{a_2(x,t)}{K\sqrt{\theta(x,t)}} + \frac{a_3(x,t)}{K\sqrt{\theta(x,t)}} \right] \quad (j=1,2), \\ \tilde{c}_{2j}(x,t) &= -\frac{1}{2} \left[ \frac{a_2(x,t)}{K\sqrt{\theta(x,t)}} - \frac{a_3(x,t)}{K\sqrt{\theta(x,t)}} \right] \quad (j=1,2), \quad \tilde{F}(x,t) = \begin{pmatrix} \frac{f(x,t)}{2K\sqrt{\theta(x,t)}} \\ f(x,t) \\ -\frac{f(x,t)}{2K\sqrt{\theta(x,t)}} \end{pmatrix}. \end{aligned}$$

Note that all conditions of Theorem 1 are fulfilled. From the condition *b*) and *b*) after simple transformations it follows,

$$\frac{a_1(x,t)}{\theta(x,t)} \in C_*(\bar{\Omega}, R) \tag{10}$$

which implies, taking *d*) into account that the function  $\tilde{C}(x,t)\Psi(t) + \tilde{F}(x,t)$  belongs to the space  $C_*(\bar{\Omega}, R^2)$ .

Then it follows from (9) by Theorem 4 [23] that the function  $\eta(x,t) = K\sqrt{\theta(x,t)}\tilde{V}^{(0)}(x,t)$  will also belong to  $C_*(\bar{\Omega}, R^2)$ .

The function  $\eta(x,t)$  satisfies the equation

$$\eta_t(x,t) = \tilde{A}(x,t)\eta + K\sqrt{\theta(x,t)}\tilde{C}(x,t)\Psi(t) + K\sqrt{\theta(x,t)}\tilde{F}(x,t) + \frac{K[a_1(x,t+d) - a_1(x,t)]}{2d\sqrt{\theta(x,t)}}\tilde{V}^{(0)}(x,t). \tag{11}$$

From (10) and the conditions of the theorem, we obtain that the function

$$K\sqrt{\theta(x,t)}\tilde{C}(x,t)\Psi(t) + K\sqrt{\theta(x,t)}\tilde{F}(x,t) + \frac{K[a_1(x,t+d) - a_1(x,t)]}{2d\sqrt{\theta(x,t)}}\tilde{V}^{(0)}(x,t) \tag{12}$$

belongs to space  $C_*(\bar{\Omega}, R^2)$ . Then from (11) by Theorem 4 from [23] it follows

$$W^{(0)}(x,t) = \sqrt{\theta(x,t)}\eta(x,t) = K\theta(x,t)\tilde{V}^{(0)}(x,t) \in C_*(\bar{\Omega}, R^2).$$

Taking into account (11), the function  $W^{(0)}(x,t)$  satisfies the equation

$W_t^{(0)}(x,t) = \tilde{A}(x,t)W + K\tilde{C}(x,t)\theta(x,t)\Psi(t) + K\theta(x,t)\tilde{F}(x,t) + K\theta_t(x,t)\tilde{V}^{(0)}(x,t)$ . Under the conditions of the theorem, the matrix  $\tilde{A}(x,t) = (\tilde{a}_{ij}(x,t))_{i,j=1}^2$  has a diagonal dominance in rows with the function  $\tilde{\theta}(x,t) = \frac{a_1(x,t)}{K\sqrt{\theta(x,t)}}$ , i.e.

$$|a_{ii}(x,t)| \geq \sum_{\substack{j=1 \\ j \neq i}}^2 |a_{ij}(x,t)| + \tilde{\theta}(x,t), \quad i=1,2,$$

where  $\tilde{\theta}(x,t) \geq \theta_0 > 0$  – is a  $\bar{\Omega}$  continuous function,  $\theta_0$  – const.

Taking into account the conditions of the theorem, it is easy to prove that the ratio of the sum of the last three terms on the right-hand side of (12) to the value of the diagonal dominance  $\tilde{\theta}(x,t)$  belongs to  $C_*(\bar{\Omega}, R^2)$ .

Then the results of [36] imply the existence of a unique solution

$W^{(0)}(x,t) = (w_1^{(0)}(x,t), w_2^{(0)}(x,t)) \in C_*(\bar{\Omega}, R^2)$  to (12) and the inequality

$$\|W^{(0)}(x,\cdot)\|_1 \leq C_1. \tag{13}$$

$C_1$  depends on the norms of the functions  $f, \psi$ , of constants  $\gamma, K, c, d, \varepsilon$ .

By virtue of (8), the coordinates  $v_i^{(0)}(x, t)$ ,  $i = 1, 2$  of the vector  $V^{(0)}(x, t)$  are represented by the formulas

$$v_1^{(0)}(x, t) = \frac{1}{K\theta(x, t)} [w_1^{(0)}(x, t) + w_2^{(0)}(x, t)], v_2^{(0)}(x, t) = \frac{1}{\sqrt{\theta(x, t)}} [w_1^{(0)}(x, t) - w_2^{(0)}(x, t)]. \quad (14)$$

Since  $W^{(0)}(x, t) \in C_*(\bar{\Omega}, R^2)$ , taking into account (10), we have

$a_1(x, t)v_1^{(0)}(x, t)$ ,  $a_0(x, t)v_2^{(0)}(x, t) \in C_*(\bar{\Omega}, R^2)$ . Using the conditions  $b)$ ,  $d)$  of Theorem 1 and inequality (13), we obtain the estimates

$$\|a_1(x, \cdot)v_1^{(0)}(x, \cdot)\|_1 \leq c_1, \quad \|a_0(x, t)v_2^{(0)}(x, t)\|_1 \leq c_2. \quad (15)$$

Here  $c_1, c_2$  are constants that depend only on the norms of the functions  $f, \psi$ , of constants  $\gamma, K, c, d, \varepsilon$ .

Substituting (14) into relation (7), we find a function  $U^{(0)}(x, t) = (u_1^{(0)}(x, t), u_2^{(0)}(x, t))$ , its belonging to the space  $C_*(\bar{\Omega}, R^2)$  is ensured by

**Theorem 1.** For functions  $a_3(x, t)u_1^{(0)}(x, t)$ ,  $a_2(x, t)u_2^{(0)}(x, t)$ , the following relations hold:

$$\begin{aligned} a_3(x, t)u_1^{(0)}(x, t) &= a_3(x, t)\psi(t) + \int_0^x a_3(x, t) \frac{w_1^{(0)}(\xi, t) + w_2^{(0)}(\xi, t)}{K\theta(\xi, t)} d\xi, \\ a_2(x, t)u_2^{(0)}(x, t) &= a_2(x, t)\dot{\psi}(t) + \int_0^x a_2(x, t) \frac{w_1^{(0)}(\xi, t) - w_2^{(0)}(\xi, t)}{K\theta(\xi, t)} d\xi. \end{aligned} \quad (16)$$

Dividing the segment  $[0, \omega]$  into parts of the same length  $\frac{\delta}{2}$ , less in view of the condition  $c)$  of

Theorem 1, we have  $\sup_{x_1, x_2 \in [0, \omega]} \frac{a_1(x_1, t)}{a_1(x_2, t)} \leq (1 + \varepsilon)^{\frac{4\omega}{\delta}} = \gamma_0$ . From this and from the notation  $\theta(x, t)$  it

follows that the expression is finite  $\sup_{x_1, x_2 \in [0, \omega]} \frac{\theta(x_2, t)}{\theta(x_1, t)}$ .

Therefore, according to the conditions of the theorem, the functions

$$\begin{aligned} a_3(x, t)\psi(t) &\equiv \frac{a_3(x, t)}{\sqrt{a_1(x, t)}} \cdot \frac{\sqrt{a_1(x, t)}}{\sqrt{\theta(x, t)}} \cdot \sqrt{\theta(x, t)}\psi(t) \text{ and} \\ \frac{a_3(x, t)}{K\theta(\xi, t)} &\equiv \frac{a_3(x, t)}{K\sqrt{a_1(x, t)}} \cdot \frac{1}{\sqrt{a_1(x, t)}} \cdot \frac{a_1(x, t)}{\theta(x, t)} \cdot \frac{\theta(x, t)}{\theta(\xi, t)}, \quad \xi \in [0, \omega], \end{aligned}$$

belong to space  $C_*(\bar{\Omega}, R^2)$ . Then from (16) we have  $a_3(x, t)u_1^{(0)}(x, t) \in C_*(\bar{\Omega}, R^2)$ .

In the same way one can show that  $a_2(x, t)u_2^{(0)}(x, t) \in C_*(\bar{\Omega}, R^2)$ . Using them and (13), from (16) we obtain the validity of the inequalities:  $\|a_3(x, \cdot)u_1^{(0)}(x, \cdot)\|_1 \leq c_3$ ,  $\|a_2(x, \cdot)u_2^{(0)}(x, \cdot)\|_1 \leq c_4$ . Here  $c_3, c_4$  are constants that depend only on the norms of the functions  $f, \psi$ , of constants  $\gamma, K, c, d, \varepsilon$ .

We put  $W^{(k)}(x, t) = K\theta(x, t)\tilde{V}^{(k)}(x, t)$  ( $k = 1, 2, \dots$ ), which is the  $\tilde{V}^{(k)} - k$  - approximation of problems (4) and (5). The fact that functions  $W^{(k)}(x, t)$  ( $k = 1, 2, \dots$ ) belong to a space  $C_*(\bar{\Omega}, R^2)$  is proved similarly to the case  $k = 0$ , based on the results of [36]. The function  $W^{(k)}(x, t)$  ( $k = 1, 2, \dots$ ) satisfies the system  $W_t^{(k)}(x, t) = \tilde{A}(x, t)W + K\tilde{C}(x, t)\theta(x, t)U^{(k-1)}(x, t) + K\theta(x, t)\tilde{F}(x, t) + K\theta_t(x, t)\tilde{V}^{(k)}(x, t)$ . Using the same approach as used for  $k = 0$ , the belonging of

functions  $a_1(x,t)v_1^{(k)}(x,t)$ ,  $a_0(x,t)v_2^{(k)}(x,t)$ ,  $a_3(x,t)u_1^{(k)}(x,t)$  and  $a_2(x,t)u_2^{(k)}(x,t)$  space  $C_*(\bar{\Omega}, R^2)$  are established.

The sequence  $\{W^{(k)}(x,t)\}_{k=1}^{\infty}$  converges in the norm of the space  $C_*(\bar{\Omega}, R^2)$  to the function  $W^{(*)}(x,t)$ , which is proved in the same way as in Theorem 1. Therefore, representations similar to (16) for sequences

$\{a_1(x,t)v_1^{(k)}(x,t)\}_{k=1}^{\infty}$ ,  $\{a_0(x,t)v_2^{(k)}(x,t)\}_{k=1}^{\infty}$ ,  $\{a_3(x,t)u_1^{(k)}(x,t)\}$ ,  $\{a_2(x,t)u_2^{(k)}(x,t)\}_{k=1}^{\infty}$  imply their convergence to functions  $a_1(x,t)v_1^{(*)}(x,t)$ ,  $a_0(x,t)v_2^{(*)}(x,t)$ ,  $a_3(x,t)u_1^{(*)}(x,t)$ ,  $a_2(x,t)u_2^{(*)}(x,t)$ , respectively, and for the  $W^{(*)}(x,t)$  inequality holds:

$$\|W^{(*)}(x,\cdot)\|_1 \leq C_2. \quad (17)$$

$C_2$  depends on the norms of the functions  $f, \psi$ , of constants  $\gamma, K, c, d, \varepsilon$ .

The function  $(v_1^{(*)}(x,t), v_2^{(*)}(x,t))$  – is the solution to problem (4), and  $(u_1^{(*)}(x,t), u_2^{(*)}(x,t))$  through them is determined by (5). Hence, taking into account the replacements made by us, we have  $u_{xif} \in C_*(\bar{\Omega}, R)$ . Using (17), we establish the boundedness of the norms  $\|a_1(x,\cdot)v_1^{(*)}(x,\cdot)\|_1$ ,  $\|a_0(x,\cdot)v_2^{(*)}(x,\cdot)\|_1$ ,  $\|a_3(x,\cdot)u_1^{(*)}(x,\cdot)\|_1$ ,  $\|a_2(x,t)u_2^{(*)}(x,t)\|_1$ .

The coercive estimate (\*) follows from this and equation (1). The theorem is proved.

This work was supported by project AR05131649 of the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan.

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#### **Оспанов М.Н.**

##### **Үшінші ретті псевдопараболалық тендеу шешімінің бір қасиеті туралы**

**Аңдатпа:** Үшінші ретті дербес дифференциалдық тендеудің ең жоғары туындысының шектелуі үшін жеткілікті жағдайлар белгіленіп,  $C_*(\bar{\Omega}, R)$  кеңістігі нормасында коэрцитивті бағасы алынды.

**Түйінді сөздер:** Псевдопараболалық тендеулер, диагональдық басымдылық, коэрцитивті баға

#### **Оспанов М.Н.**

##### **Об одном свойстве решения псевдопараболического уравнения третьего порядка**

**Аннотация.** Установлены достаточные условия ограниченности старшей производной уравнения с частными производными третьего порядка и получена коэрцитивная оценка в норме пространства  $C_*(\bar{\Omega}, R)$ .

**Ключевые слова:** псевдопараболические уравнения, диагональное преобладание, коэрцитивная оценка

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