Тлеулесова А.Б.

Импульстік әсері бар сызықты екінүктелі шеттік есептің бірмәндік шешілімдігі туралы

Аңдатпа: Импульстік әсері бар жай дифференциалдық теңдеулер жүйесі үшін шеттік есеп қарастырылады. Ұсынылып отырған есептің бірмәндік шешілімдігінің қажетті және жеткілікті шарттары тағайындалған.

Түйінді сөздер: шеттік есеп, импульс, параметрлеу тәсілі, бірмәнді шешілімділік

Тлеулесова А.Б.

О корректной разрешимости линейной двухточечной краевой задачи с импульсным воздействием

Аннотация. Рассматривается краевая задача для системы обыкновенных дифференциальных уравнений с импульсным воздействием. Установлены необходимые и достаточные условия корректной разрешимости рассматриваемой задачи.

Ключевые слова: краевая задача, импульс, метод параметризации, корректная разрешимость

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УДК 517.956

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ON ONE APPROACH TO SOLVE A NONLOCAL PROBLEM WITH PARAMETER FOR A SECOND ORDER PARTIAL INTEGRO-DIFFERENTIAL EQUATION OF HYPERBOLIC TYPE

Abstract. *A linear nonlocal problem with a parameter for partial integro-differential equations of hyperbolic type is considered. This problem is investigated by the Dzhumabaev parameterization method. We offer an algorithm for solving nonlocal problems with parameter for partial integro-differential equations of hyperbolic type. First, the original problem is reduce to an equivalent problem consisting a family of boundary value problems for ordinary integro-differential equations with parameters and integral relations. Then, we reduced the family of boundary value problems for ordinary integro-differential equations with parameters to a family of special Cauchy problems for ordinary integro-differential equations with parameters in subdomains and functional relations. At fixed values of parameters the family of special Cauchy problems for ordinary integro-differential equations in subdomains has a unique solution. A system of linear functional equations with respect to parameters is compiled. We propose an algorithm for finding an approximate solution to the equivalent problem. This algorithm includes the approximate solution of the family of Cauchy problems for ordinary differential equations and solving the linear system of functional equations.*

Key words: *nonlocal problem with parameters, partial integro-differential equations of hyperbolic type, family of boundary value problems with parameter, ordinary integro-differential equations, Dzhumabaev parameterization method, algorithm.*

The problem of constructing effective mathematical models finds its solution in many areas of life sciences and technology. A modern approach in the theory of control and identification of parameters should be connected to the development of new constructive methods and modifications of known methods for solving nonlocal problems with parameter for partial integro-differential equations of hyperbolic type. The theory of nonlocal problems with parameters for partial integro-differential equations of hyperbolic type is developing intensively and is used in various fields of biomedicine, chemistry, biology, etc. [1-12]. In spite of this, the questions of establishing the coefficient criterions of a unique solvability and constructing the approximate algorithms for finding the solutions of nonlocal problems with parameter for the partial integro-differential equations of hyperbolic type still remain open. One of the constructive methods for investigating and solving the problems with parameters for the differential equations is the Dzhumabaev parameterization method [13]. The Dzhumabaev parameterization method was developed for investigating and solving the boundary value problems for the system of ordinary differential equations. On the basis of this method, coefficient criteria for the unique solvability of linear boundary value problems for the system of ordinary differential equations were obtained. Algorithms for finding the approximate solutions were also proposed and their convergence to the exact solution of the problem studied was established. Later, the parameterization method was developed for the two-point boundary value problems for the Fredholm integro-differential equations [14-25]. Necessary and sufficient conditions for the solvability and unique solvability are established, the algorithms for finding the approximate solutions of the problems considered are constructed. In [18], methods for solving the linear boundary value problems for the Fredholm integro-differential equation on the basis of new algorithms of the parameterization method are offered. In [21] these methods are used to solve a nonlocal problem for a system of loaded and integro-differential equations of hyperbolic type.

In the present paper we propose a new approach based on the Dzhumabaev parameterization method for solving a nonlocal problem with parameters for partial integro-differential equations of hyperbolic type. We offer an approximate method to solve a nonlocal problem with parameter for partial integro-differential equations of hyperbolic type.

On the domain $\Omega=[0,T]\times[0,\omega]$ consider the linear nonlocal problem with parameters for the second order partial integro-differential equations of hyperbolic type

$$\frac{\partial^2 u}{\partial t\partial x}=A(t,x)\frac{\partial u}{\partial x}+B(t,x)\frac{\partial u}{\partial t}+C(t,x)u+\int_0^T K(t,x)\frac{\partial u(t,x)}{\partial x}dt+D(t,x)\mu(x)+f(t,x), \quad (1)$$

$$P(x)\frac{\partial u(0,x)}{\partial x}+S(x)\frac{\partial u(T,x)}{\partial x}=\varphi_1(x), \quad x\in[0,\omega], \quad (2)$$

$$u(t,0)=\psi(t), \quad t\in[0,T], \quad (3)$$

$$\frac{\partial u(\theta,x)}{\partial x}=\varphi_2(x), \quad x\in[0,\omega], \quad (4)$$

where $u(t,x)$ is an unknown function, $\mu(x)$ is an unknown functional parameter, the functions $A(t,x)$, $B(t,x)$, $C(t,x)$, $K(t,x)$, $D(t,x)$, and $f(t,x)$ are continuous on Ω, the functions $P(x)$, $S(x)$, $\varphi_1(x)$, $\varphi_2(x)$ are continuous on $[0,\omega]$, and the function $\psi(t)$ is continuously differentiable on $[0,T]$.

Let $C(\Omega,R)$ ($C([0,\omega],R)$) denote the space of continuous functions $u:\Omega\to R$ ($\mu:[0,\omega]\to R$) with the norm $\|u\|_1=\max_{(t,x)\in\Omega}\|u(t,x)\|$ ($\|\mu\|_1=\max_{x\in[0,\omega]}\|\mu(x)\|$).

A solution to problems (1)-(4) is a pair $(u^*(t,x),\mu^*(x))$, with $u^*(t,x)\in C(\Omega,R)$, $\mu^*(x)\in C([0,\omega],R)$, where the function $u^*(t,x)$ has partial derivatives $\frac{\partial u^*(t,x)}{\partial x}\in C(\Omega,R)$, $\frac{\partial u^*(t,x)}{\partial t}\in C(\Omega,R)$, $\frac{\partial^2 u^*(t,x)}{\partial t\partial x}\in C(\Omega,R)$ and satisfies the partial integro-differential equation of hyperbolic type (1) with $\mu(x)=\mu^*(x)$ and boundary conditions (2), (3) and (4).

Introduce the new functions $v(t,x)=\dfrac{\partial u(t,x)}{\partial x}$, $w(t,x)=\dfrac{\partial u(t,x)}{\partial t}$.

We reduce problems (1)-(4) to an equvalent problem

$$\frac{\partial v}{\partial t}=A(t,x)v+\int_0^T K(t,x)v(t,x)dt+D(t,x)\mu(x)+B(t,x)w(t,x)+C(t,x)u(t,x)+f(t,x), \quad (5)$$

$$P(x)v(0,x)+S(x)v(T,x)=\varphi_1(x), \quad x\in[0,\omega], \quad (6)$$

$$v(\theta,x)=\varphi_2(x), \quad x\in[0,\omega], \quad (7)$$

$$u(t,x)=\psi(t)+\int_0^x v(t,\xi)d\xi, \qquad w(t,x)=\dot{\psi}(t)+\int_0^x \frac{\partial v(t,\xi)}{\partial t}d\xi, \quad (8)$$

where condition (3) takes account into relations (8).

A solution to problems (5)-(8) is a quadruple $(v^*(t,x),\mu^*(x),u^*(t,x),w^*(t,x))$, with $v^*(t,x)\in C(\Omega,R)$, $\mu^*(x)\in C([0,\omega],R)$, $u^*(t,x)\in C(\Omega,R)$, $w^*(t,x)\in C(\Omega,R)$, where the function $v^*(t,x)$ has partial derivative $\dfrac{\partial v^*(t,x)}{\partial t}\in C(\Omega,R)$ and satisfies the integro-differential equation (5) for all $(t,x)\in\Omega$ with $\mu(x)=\mu^*(x)$, $u(t,x)=u^*(t,x)$, $w(t,x)=w^*(t,x)$ and boundary conditions (6) and (7), here the functions $u^*(t,x)$ and $w^*(t,x)$ are connected with functions $v^*(t,x)$ and $\dfrac{\partial v^*(t,x)}{\partial t}$ by integral relations (8).

Further, we apply the Dzhumabaev parameterizaion method.

Given the points: $t_0=0<t_1<t_2<\ldots<t_m=\theta<\ldots<t_{N-1}<t_N=T$, and let $\Delta_N(\theta,\omega)$ be the partition of domain Ω into N subdomains: $\Omega=\bigcup_{r=1}^{N}\Omega_r$, $\Omega_r=[t_{r-1},t_r)\times[0,\omega]$, $r=1,2,...,N-1$, $\Omega_N=[t_{N-1},t_N]\times[0,\omega]$.

By $C(\Omega,\Delta_N(\theta,\omega),R^N)$ we denote the space of function systems $v([t],x)=(v_1(t,x),v_2(t,x),\ldots,v_N(t,x))$, where $v_r:\Omega_r\to R$ are continuous and have finite left-hand limits $\lim_{t\to t_r-0}v_r(t,x)$ for all $r=1,2,...,N$, with the norm $\|v\|_2=\max_{r=\overline{1,N}}\sup_{t\in\Omega_r}\|v_r(t,x)\|$.

Denote by $v_r(t,x)$ the restriction of function $v(t,x)$ to the r-th domain Ω_r and reduce problems (1)-(4) to the equivalent family of multipoint problems with parameter for the ordinary integro-differential equations

$$\frac{\partial v_r}{\partial t}=A(t,x)v_r+\sum_{j=1}^{N}\int_{t_{j-1}}^{t_j}K(t,x)v_j(t,x)dt+D(t,x)\mu(x)+B(t,x)w(t,x)+C(t,x)u(t,x)+f(t,x), \quad (9)$$

$$r=1,2,...,N,\ P(x)v_1(0,x)+S(x)v_N(T,x)=\varphi_1(x), \quad x\in[0,\omega], \quad (10)$$

$$v_{m+1}(\theta,x)=\varphi_2(x), \quad x\in[0,\omega], \quad (11)$$

$$\lim_{t\to t_p-0}v_p(t,x)=v_{p+1}(t_p,x), \qquad p=1,2,...,N-1, \quad (12)$$

$$u(t,x)=\psi(t)+\int_0^x v_r(t,\xi)d\xi, \quad w(t,x)=\dot{\psi}(t)+\int_0^x\frac{\partial v_r(t,\xi)}{\partial t}d\xi, \quad (t,x)\in\Omega_r,\ r=1,2,...,N, \quad (13)$$

where (12) are conditions for matching the solution at the interior points of partition $\Delta_N(\theta,\omega)$.

The solution of problems (9)-(13) is a quadruple $(v^*([t],x),\mu^*(x),u^*(t,x),w^*(t,x))$ with elements $v^*([t],x)=(v_1^*(t,x),v_2^*(t,x),\ldots,v_N^*(t,x))\in C(\Omega,\Delta_N(\theta,\omega),R^N)$, $\mu^*(x)\in C([0,\omega],R)$ $u^*(t,x)\in C(\Omega,R)$, $w^*(t,x)\in C(\Omega,R)$, where functions $v_r^*(t,x)$, $r=1,2,...,N$, are continuously differentiable on Ω_r, which satisfies system of integro-differential equations (9) with $\mu(x)=\mu^*(x)$,

$u(t,x)=u^*(t,x)$, $w(t,x)=w^*(t,x)$, and boundary conditions (10) and (11) and continuity conditions (12), the functions $u^*(t,x)$ and $w^*(t,x)$ are connected with functions $v_r^*(t,x)$ and $\frac{\partial v_r^*(t,x)}{\partial t}$ by integral relations (13) for $(t,x)\in\Omega_r$, $r=1,2,...,N$.

We introduce additional parameters $\lambda_r(x)=v_r(t_{r-1},x)$, $r=1,2,...,N$, and $\lambda_{N+1}(x)=\mu(x)$. Making the substitution $\tilde v_r(t,x)=v_r(t,x)-\lambda_r(x)$, on every $r-$th domain Ω_r, $r=1,2,...,N$, we obtain the family of multipoint problems with parameters

$$\frac{\partial\tilde v_r}{\partial t}=A(t,x)\tilde v_r+A(t,x)\lambda_r(x)+\sum_{j=1}^{N}\int_{t_{j-1}}^{t_j}K(t,x)[\tilde v_j(t,x)+\lambda_j(x)]dt+D(t,x)\lambda_{N+1}(x)+$$

$$+B(t,x)w(t,x)+C(t,x)u(t,x)+f(t,x),\quad (t,x)\in\Omega_r,\ r=1,2,...,N, \tag{14}$$

$$\tilde v_r(t_{r-1},x)=0,\qquad r=1,2,...,N, \tag{15}$$

$$P(x)\lambda_1(x)+S(x)\lambda_N(x)=\varphi_1(x)-S(x)\tilde v_N(T,x),\quad x\in[0,\omega], \tag{16}$$

$$\lambda_{m+1}(x)=\varphi_2(x),\quad x\in[0,\omega], \tag{17}$$

$$\lambda_p(x)-\lambda_{p+1}(x)=-\lim_{t\to t_p-0}\tilde v_p(t,x)\quad,\quad p=1,2,...,N-1, \tag{18}$$

$$u(t,x)=\psi(t)+\int_0^x[\tilde v_r(t,\xi)+\lambda_r(x)]d\xi,\quad w(t,x)=\dot\psi(t)+\int_0^x\frac{\partial\tilde v_r(t,\xi)}{\partial t}d\xi,\quad (t,x)\in\Omega_r,\ r=1,2,...,N, \tag{19}$$

A quadruple $(\tilde v^*([t],x),\lambda^*(x),u^*(t,x),w^*(t,x))$ with elements $\lambda^*(x)=(\lambda_1^*(x),\lambda_2^*(x),\ldots,\lambda_{N+1}^*(x))$, $\lambda_r^*(x)\in C([0,\omega],R)$, $r=1,2,...,N+1$, $\tilde v^*([t],x)=(\tilde v_1^*(t,x),\tilde v_2^*(t,x),\ldots,\tilde v_N^*(t,x))\in C(\Omega,\Delta_N(\theta,\omega),R^N)$, $u^*(t,x)\in C(\Omega,R)$, $w^*(t,x)\in C(\Omega,R)$, is said to be a solution to problems (14)-(19) if the functions $\tilde v_r^*(t,x)$, $r=1,2,...,N$, are continuously differentiable on Ω_r and satisfy (14) with $\lambda_r(x)=\lambda_r^*(x)$, $r=1,2,...,N+1$, $u(t,x)=u^*(t,x)$, $w(t,x)=w^*(t,x)$, and initial conditions (15), boundary conditions (16) and (17) and continuity conditions (18), the functions $u^*(t,x)$ and $w^*(t,x)$ are connected with functions $\tilde v_r^*(t,x)$ and $\frac{\partial\tilde v_r^*(t,x)}{\partial t}$ by integral relations (19) for $(t,x)\in\Omega_r$, $r=1,2,...,N$.

At fixed $\lambda_r(x)$, $u(t,x)$, $w(t,x)$ problems (14) and (15) are a family of special Cauchy problems for integro-differential equations, where $r=1,2,...,N+1$. The variable x changes on $[0,\omega]$ and plays role of the parameter of the family.

For fixed $x\in[0,\omega]$ and $\lambda_r(x)$, $u(t,x)$, $w(t,x)$, we have a special Cauchy problem for integro-differential equations. This problem is studied in [14-17]. Conditions of unique solvability are established in the terms of initial data.

Algorithm. The unknown function $\tilde v([t],x)$ will be determined from family of special Cauchy problems for integro-differential equations (14) and (15). The unknown parameters $\lambda_r(x)$, $r=1,2,...,N+1$ will be found from functional equations (16)-(18). The unknown functions $u(t,x)$ and $w(t,x)$ will be found from integral relations (19).

If we know the functions $\lambda_r(x)$, $r=1,2,...,N+1$, $u(t,x)$, $w(t,x)$, then from family of special Cauchy problem for integro-differential equations (14) and (15) we find the function $\tilde v([t],x)$. Conversely, if we know the function $v([t],x)$, then from functional equations (16)-(18) we find the functions $\lambda_r(x)$, $r=1,2,...,N+1$. Further, using founded functions, from integral relations we determine $u(t,x)$ and $w(t,x)$.

Since the functions $v([t],x)$, $\lambda_r(x)$, $u(t,x)$ and $w(t,x)$ are unknown, for finding a solution to problems (14)--(19) we use an iterative method. The solution to problems (14)--(19) is a quadruple of functions $(\tilde{v}^*([t],x),\lambda^*(x),u^*(t,x),w^*(t,x))$ which we define as the limit of the sequence of quadruples $(\tilde{v}^{(k)}([t],x),\lambda^{(k)}(x),u^{(k)}(t,x),w^{(k)}(t,x))$, $k=0,1,2,...$ according to the following algorithm:

Step 0. 1) Setting $\tilde{v}_s(t,x)=0$, $(t,x)\in\Omega_s$, $s=1,2,...,N$, in the right-hand part of the system of functional equations (16)-(19), we find the initial approximation $\lambda_r^{(0)}(x)$ for all $x\in[0,\omega]$, $r=1,2,...,N+1$;

2) In the right-hand part of the system, setting $\lambda_r(x)=\lambda_r^{(0)}(x)$, $r=1,2,...,N+1$, $u(t,x)=\psi(t)$, $w(t,x)=\dot{\psi}(t)$, from the family of special Cauchy problem for integro-differential equations we find the initial approximations $\tilde{v}_s^{(0)}(t,x)$ for $(t,x)\in\Omega_s$, $s=1,2,...,N$;

3) From integral relations (19) under $v_s(t,x)=v_s^{(0)}(t,x)$, $(t,x)\in\Omega_s$, $s=1,2,...,N$, $\lambda_r(x)=\lambda_r^{(0)}(x)$, $r=1,2,...,N$, we find the function $u^{(0)}(t,x)$ and $w^{(0)}(t,x)$ for all $(t,x)\in\Omega$.

Step 1. 1) Suppose in the right-hand part of the system of functional equations (16)-(19) $\tilde{v}_s(t,x)=\tilde{v}_s^{(0)}(t,x)$, $(t,x)\in\Omega_r$, $s=1,2,...,N$, from system (16)--(19) we find the first approximation $\lambda_r^{(1)}(x)$ for all $x\in[0,\omega]$, $r=1,2,...,N+1$;

2) Suppose in the right-hand part of the system $\lambda_r(x)=\lambda_r^{(1)}(x)$, $r=1,2,...,N+1$, $u(t,x)=u^{(0)}(t,x)$, $w(t,x)=w^{(0)}(t,x)$, from the family of special Cauchy problems for integro-differential equations we find the first approximations $\tilde{v}_s^{(1)}(t,x)$ for $(t,x)\in\Omega_s$, $s=1,2,...,N$;

3) From integral relations (19) under $v_s(t,x)=v_s^{(1)}(t,x)$, $s=1,2,...,N$, $\lambda_r(x)=\lambda_r^{(1)}(x)$, $r=1,2,...,N$, we find the function $u^{(1)}(t,x)$ and $w^{(1)}(t,x)$ for all $(t,x)\in\Omega$.

And so on.

Step k. 1) Suppose in the right-hand part of the system of functional equations (16)-(19) $\tilde{v}_s(t,x)=\tilde{v}_s^{(k-1)}(t,x)$, $(t,x)\in\Omega_r$, $s=1,2,...,N$, from systems (16)--(19) we find the kth approximation $\lambda_r^{(k)}(x)$ for all $x\in[0,\omega]$, $r=1,2,...,N+1$;

2) Suppose in the right-hand part of the system $\lambda_r(x)=\lambda_r^{(k)}(x)$, $r=1,2,...,N+1$, $u(t,x)=u^{(k-1)}(t,x)$, $w(t,x)=w^{(k-1)}(t,x)$, from the family of special Cauchy problem for integro-differential equations we find the kth approximations $\tilde{v}_s^{(k)}(t,x)$ for $(t,x)\in\Omega_s$, $s=1,2,...,N$;

3) From integral relations (19) under $v_s(t,x)=v_s^{(k)}(t,x)$, $s=1,2,...,N$, $\lambda_r(x)=\lambda_r^{(k)}(x)$, $r=1,2,...,N$, we find the function $u^{(k)}(t,x)$ and $w^{(k)}(t,x)$ for all $(t,x)\in\Omega$.

$k=1,2,3,...$.

Conditions of feasibility and convergence of the constructed algorithm and the conditions of the existence of a unique solution to problems (14)-(19) are established.

For obtaining conditions of the unique solvability to original problems (1)-(4) we use results obtained in [26-31].

***Funding**: The work is supported by the grants (№№ AP05132486, AP05131220, AP05132455) of Ministry of Education and Science of the Republic of Kazakhstan (2018-2020).*

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Екінші ретті гиперболалық тектес дербес туындылы интегралдық-дифференциалдық теңдеулер үшін параметрі бар бейлокал есепті шешуге арналған бір тәсіл туралы

Аңдатпа: Екінші ретті гиперболалық тектес дербес туындылы интегралдық-дифференциалдық теңдеулер үшін параметрі бар сызықты бейлокал есеп қарастырылады. Осы есеп Жұмабаевтың параметрлеу әдісімен зерттеледі. Біз гиперболалық тектес дербес туындылы интегралдық-дифференциалдық теңдеулер үшін параметрі бар сызықты бейлокал есепті шешудің алгоритмін ұсынамыз. Біріншіден, бұл есеп параметрлері бар жай интегралдық-дифференциалдық теңдеулер үшін шеттік есептер әулеті мен интегралдық қатынастардан тұратын пара-пар есепке келтіріледі. Содан кейін, параметрлері бар жай интегралдық-дифференциалдық теңдеулер үшін шеттік есептер әулеті ішкі облыстардағы параметрлері бар жай интегралдық-дифференциалдық теңдеулер үшін арнайы Коши есептері әулетіне және функционалдық қатынастарға келтіріледі. Параметрлердің бекітілген мәндерінде ішкі облыстардағы параметрлері бар жай интегралдық-дифференциалдық теңдеулер үшін арнайы Коши есептері әулетінің жалғыз шешімі бар болады. Параметрлерге қатысты сызықты функционалдық теңдеулер жүйесі құрылады. Пара-пар есептің жуық шешімін табудың алгоритмі беріледі. Бұл алгоритм жай интегралдық-дифференциалдық теңдеулер үшін арнайы Коши есептері әулетін жуықтап шешуді және сызықты функционалдық теңдеулер жүйесін шешуді қамтиды.

Түйінді сөздер: параметрі бар бейлокал есеп, гиперболалық тектес дербес туындылы интегралдық-дифференциалдық теңдеулер, параметрі бар шеттік есептер әулеті, жай интегралдық-дифференциалдық теңдеулер, Жұмабаевтың параметрлеу әдісі, алгоритм

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Об одном подходе к решению нелокальной задачи с параметром для интегро-дифференциальных уравнений в частных производных гиперболического типа второго порядка

Аннотация. Рассматривается линейная нелокальная задача с параметром для интегро-дифференциальных уравнений в частных производных гиперболического типа. Эта задача исследуется методом параметризации Джумабаева. Предлагается алгоритм решения нелокальной задачи с параметром для интегро-дифференциальных уравнений в частных производных гиперболического типа. Во-первых, исходная задача сводится к эквивалентной задаче, состоящей из семейства краевых задач для обыкновенных интегро-дифференциальных уравнений и интегральных соотношений. Затем, семейства краевых задач для обыкновенных интегро-дифференциальных уравнений сводятся к семейству специальных задач Коши для обыкновенных интегро-дифференциальных уравнений с параметрами на подобластях и функциональным соотношениям. При фиксированных значениях параметров семейство специальных задач Коши для обыкновенных интегро-дифференциальных уравнений имеет единственное решение. Составляется линейная система функциональных уравнений относительно параметров. Предлагается алгоритм нахождения приближенных решений эквивалентной задачи.

Данный алгоритм включает приближенное решение семейства специальных задач Коши для обыкновенных интегро-дифференциальных уравнений и решение линейной системы функциональных уравнений.

Ключевые слова: нелокальная задача с параметром, интегро-дифференциальные уравнения в частных производных гиперболического типа, семейства краевых задач с параметром, обыкновенные интегро-дифференциальные уравнения, метод параметризации Джумабаева, алгоритм.

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