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PREDICTIVE MODELING OF FUTURE FLOODS IN THE ALMATY REGION USING MACHINE LEARNING METHODS

The article is dedicated to my supervisor, a very kind, good person and scientist, d. phys.-math.sci., professor D.S. Dzhumabaev

Abstract. *The development of modern science and technology allows us to realize many opportunities that are not yet available. For example, the study of various natural phenomena and their hidden dangers, as well as forecasting, can prevent or intensify preventive measures. The impact, the region and frequency of their occurrence affect our lives to an unprecedented extent. It is very difficult to prevent these events in the short term, but a risk prevention plan can reduce the negative consequences of an accident. The present study is focused on the evaluation of flood potential within Malaya Almatinka river basin in Almaty using four prediction models RandomForest, LinearRegression, DecisionTree and XGBoost.*

Key words: *prediction of flood, machine learning, RandomForestRegressor, LinearRegressionRegressor, DecisionTreeRegressor and XGBoostRegressor.*

1. Introduction

Floods belong to one of the most frequent as well as devastating natural disasters worldwide [1]. Due to ongoing climate change and increasing anthropic pressure on the landscape [2], the frequency and magnitude of future flood situations is expected to rise while the development of the population’s resilience against floods is questionable, especially, in developing countries [3-5].

A lot of scientific papers are devoted to topics such as predicting flooding, for determining an area as having very low to very high flood potential, through approaches that use hydrological-hydraulic models for flood modeling. And there are also works dedicated to the study of flood susceptibility through geospatial technologies [6-15] and a lot of references to these are given in the work [16]. All flood studies were conducted using data from different countries of the world, but not from Kazakhstan.

In the summer of 1921, the Malaya Almatinka river was transported to the center of the villages. From the large stones destroyed along with the mudslide, Almaty turned into rubble and sank into the mud. More than 500 people were killed in the accident. "This was a great loss of life for a city built from the logs of single-story houses with a population of only thirty thousand people. The city was severely affected by this flood, all the streets are filled with water. In six hours, flood water delivered 7 million cubic meters of water and 3,250,000 cubic meters of rock, sand and mud to the city. In [17, p. 146-152] there is a description of the flood. The paper presents methods for the calculation of flooding zones in a territory with the use of a digital elevation model on the basis of successive pools [18,19].

In this paper we used the Malaya Almaty river basin in Almaty to study the prediction of flooding using four prediction models RandomForest, LinearRegression, DecisionTree and XGBoost. 10 flood predictors, 8 flood locations, and 8 non-flood locations were used. As input, the model used the percentage of 70% of the places where flooding and flooding occurred. Of the input data, 70% were used as a training sample, and 30% were used as a test sample. The highest accuracy was obtained by the RandomForestRegressor model in terms of testing (0.853) samples. Checking the results performed by the R2 method emphasizes that the RandomForestRegressor model gave the most accurate results.

2. Study area

Malaya Almatinka is a river in Almaty, a right tributary of the Kaskelen river. It originates from the Tuyuksu glaciers of the Zailiysky Alatau range. With a length of 125 km, it has a catchment area of 710 km^2. The main tributaries are Sarysay (Yellow Log), Kuigensay (Gorelnik), Kimasar, Zharbulak (Kazachka), Battery (Bedelbay), Butakovka, Karasu-Turksib, Esentai, Karasu and Terenkara.

Physical and geographical characteristics. The Malaya Almatinka is located in three different landscape zones: mountain, foothill and plain. The riverbed in the mountain zone is moderately meandering, composed of boulder-pebble deposits, width 3-13 m; river depth from 0.15 to 0.5 m; the average long-term annual flow of the river is 0.32 m^3/s, at the meteorological station mynzhilki, 2.3 m^3/s.

All the catastrophic floods in the twentieth century, which almost covered Almaty, were in the month of July in 1921, 1956, 1963, 1973 and 1977. In particular, these events happened on July 8th, 7th, 15th and the August 3rd [20]. In October 1966, an anti-settlement dam was built in the Medeu tract by a directional explosion in the river basin. At the exit from the Malaya Almaty gorge the river divides into 3 branches: Esentai (Vesnovka a), zharbulak (Cossack) and the Malaya Almatinka. In the city of Almaty, the Malaya Almatinka flows through the Eastern part of the city, and its banks are concreted. The river basin has 46 lakes, ponds and reservoirs with a total surface area of 2.5 km^2.

3. Data

Given the fact that the main purpose of this study is to predict future flooding, the data used in this case were taken from different sources. In the present case, the historical flood events were collected from the books and web sites [21-24]. 10 factors affecting flooding were taken such as maximum and minimum temperature, date, rainfall, slope, land use, elevation, and region and target.

Data sets consist of 1100 pieces of data. Data were taken for Big Almaty lake, Chimbulak, Kamenskoye plateau, Medeo, Alamty city, the district Airport, Issyk city, Kapchagai city etc. For the 10 factors listed above, data is collected from 1921 to 2020 for 8 locations. The flood conditioning factors are briefly described below.

Slope angle directly influences the velocity of surface runoff and water accumulation potential and, therefore, is considered one of the main factors which contributes to flood phenomena genesis. In the case of the present research, the slope angle values, range from 0^0 to 18^0. Precipitation (rainfall) is an — atmospheric phenomena associated with the presence of water in the atmosphere in a liquid or solid state, falling from clouds or deposited from the air on the Earth's surface and any objects. Precipitation is measured by the thickness of the fallen water layer in millimeters. On average, the globe receives about 1000 mm of precipitation per year, and in deserts and high latitudes, there is less than 250 mm per year. In this study, precipitation values vary from 0 to 374.

Land use is the characterization of land based on what can be built on it and what the land can be used for. It takes only 3 forms in the context of the study: mountainous, urban, and rural.

Elevation (Elevation Height) is the height above sea level or absolute height. This is the difference between a point on land and the height of the sea. It is usually calculated based on average sea level, the part of a particular area that vertically exceeds above sea level. The starting point of elevation is called the zero point of elevation or zero point of level, which is the average surface of the sea on a particular coast. This is calculated based on long-term records of the local wave station by getting the average state of the sea surface. The elevation is an important flood predictor due to the fact that it differentiates the areas located at high altitude, where phenomena are less likely than in areas located at low altitudes, which are more exposed to the flood phenomena due to the direction of water runoff from high altitudes to low altitudes [25]. We have 8 regions for research: mountain, rural and urban, and each of them has a corresponding Elevation (Elevation Height). Attribute targets consist of the two values 0 and 1, where 0 means ‘no flood’ and 1 means ‘flood present’.

All these factors are included for different times starting from 1921 to 2020 (including the month of March), but not all years and months are taken into account. The resulting dataset consists of 1100 rows and 10 attributes.

4. Methods used for predicting flooding

For the study, several machine learning algorithms were built, four of which were selected, which gave good results. The dataset was split into a training set and a test set. The model was built on 70% of the data, and checked on 30%, and to implement the algorithms we used the methods of the sklearn library on Python. We designated all of the attribute values as X, except Target, Y, which is a vector consisting of the value of the Target in the data set. Below is a diagram of the implementation of the prediction models (Fig.1).

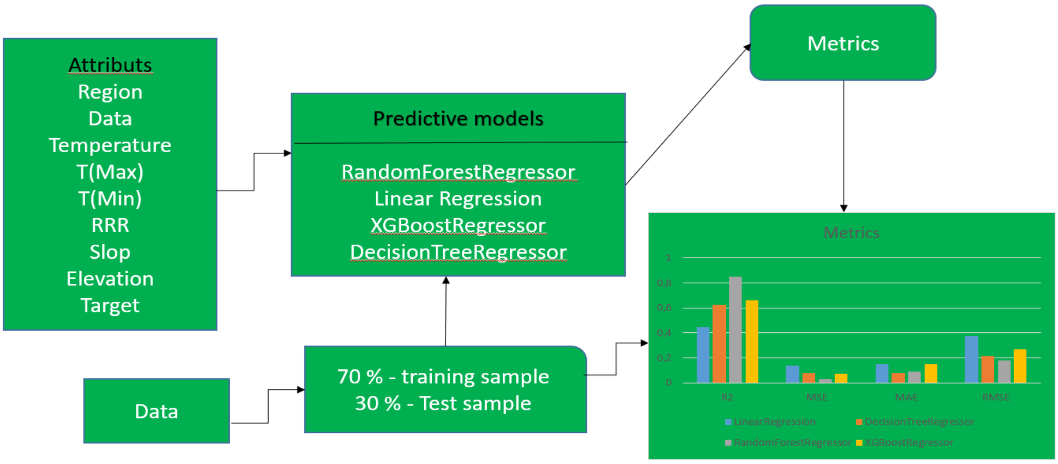

Figure-1. The scheme of prediction models

4.1 Linear Regression

The algorithms for regression are one of the types of control algorithms. An algorithm is used for building a model using data from a test suite, and then computed using test data from this model. In linear regression the target value is expected to be a linear combination of the features. In mathematical notation, if $\hat{y}(w,x) = w_0 + w_1x_1 + \ldots + w_px_p$

we designate the vector $w = (w_1, w_2, \ldots, w_p)$ as coef_ and w_0 as intercept_.

fits a linearregression model with coefficients $w = (w_1, w_2, \ldots, w_p)$ to minimize the residual sum of squares between the observed targets in the dataset, and the targets predicted by the linear approximation. Mathematically it solves a problem of the form:

$$\min_w \|xw - y\|_2^2 \tag{1}$$

LinearRegression will take in its fit method arrays X and y and will store the coefficients w of the linear model in its coef_ member. X is the matrix which consists of all attribute values, except Target, Y, which is Target for 70% of them for training and 30% for testing.

4.2 Decision Tree Regression

Regression Tree is a simple but powerful tool used to build prediction models from a large set of data, once it identifies which auxiliary variables are able to explain the variability of the response variable. The models are obtained by recursive partitioning of all the data concentrated in the root node (according to the most significant auxiliary variable) and fitting a simple prediction model within each partition [26]. According to [27], regression trees can fit almost every kind of traditional statistical model, including least squares, quantile, logistic, Poisson and proportional hazard models, as well as models for longitudinal and multiresponse data. The quality criterion in a desition tree regressor is $D = \frac{1}{l}\sum_{i=1}^{l}(y_i - \frac{1}{l}\sum_{i=1}^{l} y_i)$, where l is the number of objects in the sheet and y_i is the values of the target attribute. We minimize the variance around the average, and we look for features that break down the sample so that the values of the target feature in each sheet are approximately equal.

4.3 XGboost Regression

Through boosting - a training sample at each iteration is determined based on classification errors at previous iterations. The process of XGBoost involves assembling a base model for the pre-existing model, for example, training an initial tree, constructing a second tree combined with the initial tree, and repeating the second step until the expected number of trees is reached.

The idea of gradient boosting is to train each subsequent algorithm on a discrepancy with real answers, to move towards reducing empirical risk. Let it be $h(x,\theta)$ - based algorithms.

$$\alpha_R(x) = \sum_{i=1}^{R} \beta_i h(x,\theta)$$

$$Q = \sum_{i=1}^{N} L(\alpha, y_i)$$

Boosting step for $r = \overline{1,R}$

$$\nabla Q = \left[\frac{\partial L(\alpha_{r-1}, y_i)}{\partial \alpha_{r-1}} (x_i) \right]_{i=1}^{N}$$

$$\theta_r = learn(X, \nabla Q)$$

$$\beta_r = \arg\min_{\beta} \sum_{i=1}^{N} \left(L(\alpha_{r-1}(x_i) + \beta \cdot h(x_i, \theta_r)), y_i \right)$$

$$\alpha_r = \alpha_{r-1}(x) + \beta_r \cdot h(x, \theta_r),$$

where $\theta -$ is learning rate. In the initial step learning rate was equal to 0.1 and $\beta_0 = 0$ was chosen by default, and $L -$is loss function, which depends on the type of problem being solved. It must be differentiable, and in our case the difference is squared between observed and predicted data, such as (1).

4.4 Random Forest Regression

Random forest is a bagging technique and not a boosting technique. The trees in random forests are run in parallel. There is no interaction between these trees while building the trees.

Random Forest is a set of decision trees. In the regression problem, their responses are averaged; in the classification problem, the decision is made by majority vote. All trees are built independently according to the following scheme:

- a sub-sample of the training sample is selected and a tree is built based on it (each tree has its own sub — sample).
- to build each split in the tree, view the max_features of random features (each new split has its own random features).
- choosing the best features and splitting it. The tree is usually built before the selection is exhausted, but modern implementations have parameters that limit the height of the tree, the number of objects in the leaves, and the number of objects in the subsample at which splitting is performed.

Data for training and testing is taken as indicated above. The best random forest regression parameters are selected using the methods of the sklearn library. Below is a model with parameters in our case:

```
RandomForestRegressor(bootstrap=True, criterion='mse', max_depth=None,
                      max_features='auto', max_leaf_nodes=None,
                      min_impurity_decrease=0.0, min_impurity_split=None,
                      min_samples_leaf=1, min_samples_split=2,
                      min_weight_fraction_leaf=0.0, n_estimators=100,
                      n_jobs=None, oob_score=False, random_state=42, verbose=0,
                      warm_start=False)
```

5. Results

The results validation was made using the testing dataset, and prediction data with the help of the training dataset. Regarding the Success Rate, the Random Forest Regression was the most performant model with the R^2 equal to 0,853, MSE equal to 0,032, MAE is equal to 0,032 and RMSE equal to 0,179, followed by the XGBoost regressor. All the results of metrics are shown in Fig. 2 and Fig.3.

Metrics for evaluating model results.

MAE (Mean absolute error): $MAE = \frac{1}{n}\sum_{i=1}^{n}\left|\alpha(x_i) - y_i\right|$,

RMSE (Root mean squared error): $RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\alpha(x_i) - y_i\right)^2}$,

MSE (Mean squared error): $MSE = \frac{1}{n}\sum_{i=1}^{n}\left(\alpha(x_i) - y_i\right)^2$,

R^2 (Coefficient of determination): $R^2 = 1 - \frac{\sum_{i=1}^{N}\left(\alpha(x_i) - y_i\right)^2}{\sum_{i=1}^{N}\left(\overline{y} - y_i\right)^2}$,

often called R^2, represents the predictive power of the model as a value between 0 and 1. Zero means that the model is random (i.e. it does not explain anything); 1 means that there is a perfect fit.

Where $\alpha(x_i)$ - based algorithms, $y_{predict}(x_i) = \alpha(x_i) + \xi_i$, ξ_i is $N(0, \sigma_i)$ - normal distribution, σ is root of the variance, y_i observations, $\overline{y}$ is the observation's overall mean.

Figure 2. Metrics of models used by histogram

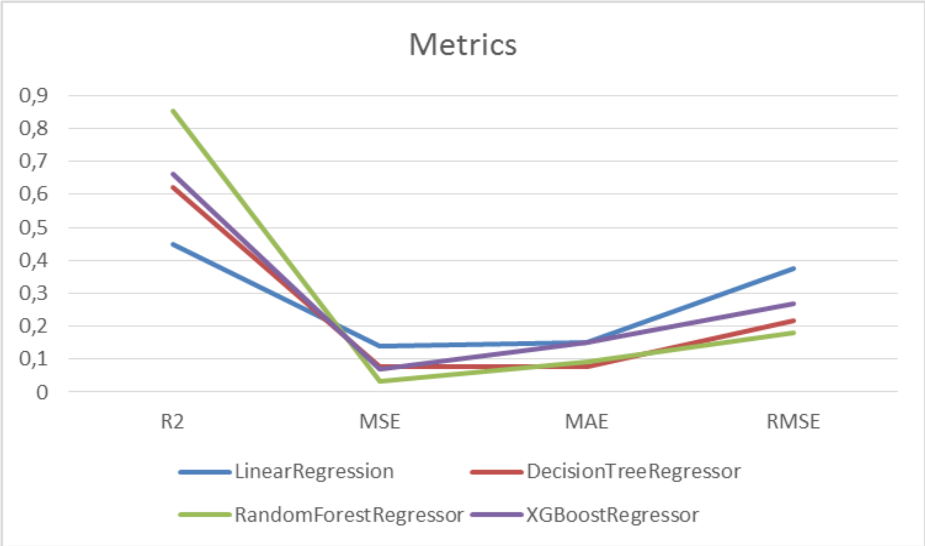

Figure 3. Metrics of models used by line

While minimizing errors and finding weights with fewer errors, we can also use a lot of methods. When we find weights so that there are minimal errors, we will use the gradient descent method, and we can also take derivatives equal to zero and obtain a system of equations for weights. To solve such systems, we can use different methods, one of which is given [28] for the general case.

6. Discussion

The Almaty region is one of the regions affected by mudflows. According to historical data, there were several large floods precisely along the Malaya Almatinka River. We are talking about mountain glaciers. Therefore, temperature and rainfall in the mountains affect the presence of floods. Below, in Fig.4, you see that one of the most important variables for the incidence of existing flood is the minimum temperature, followed by RRR meaning rainfall. This can be explained by the fact that when there are low temperatures after rain, it will snow, especially in the mountains. After a low temperature, according to statistical studies of the temperature in the districts of Almaty, there is always a high temperature, which affects the appearance of floods.

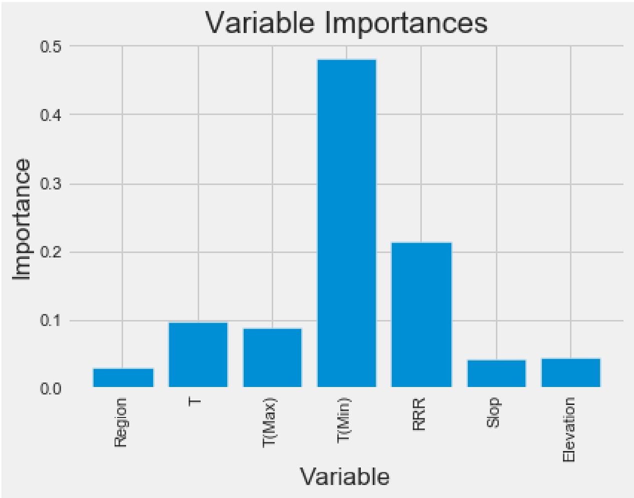

Figure 4. Feature importances

7. Conclusion

In the present study, the capability of four models was tested in terms of flood susceptibility prediction. This study comes in the context of the urgent measures, that should be taken to reduce the negative effects of floods. It should be remarked that in order to train the models and in the same time to evaluate their performance, the initially established training dataset was also divided into a sample used to train the models (70%) and another sample used to test the model performance (30%). To achieve good results, some model parameters were optimized using the 15-folds cross-validation procedure.

Thus, with the highest performance, the model built using the random forest algorithm gave the best result. The prediction using the model gave a positive result, that is, the forecast for floods in Almaty and Almaty region during the summer periods of 2020 and 2021 along the Malaya Almatinka river, will not occur with a probability of 83%.

At the moment, prediction using machine learning methods in Almaty has not been investigated. This work will be investigated in the future, supplemented with different datasets, such as satellite images, and so on.

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Кабдрахова С.С.

Алматы облысындағы болашақ су тасқындарын машиналық оқыту әдістерін пайдалана отырып болжау

Аңдатпа: Қазіргі заманғы ғылым мен техниканың дамуы бізге әлі қол жетімді емес көптеген мүмкіндіктерді жүзеге асыруға мүмкіндік береді. Мысалы, әртүрлі табиғи құбылыстар мен олардың жасырын қауіптерін зерттеу, сондай-ақ алдын ала апатты болдырмауын болжау, немесе сақтану іс әрекеттерді жасауға мүмкіндік тудырады. Аймақ, су тасқынының әсері және оның пайда болу жиілігі біздің өмірімізге бұрын-соңды болмаған дәрежеде әсер етеді. Қысқа мерзімде бұл оқиғалардың алдын алу өте қиын, бірақ тәуекелдерді болдырмау жоспары апаттың теріс салдарын азайтуы мүмкін. Бұл зерттеу жұмысы кездейсоқ орман ағашы, сызықтық регрессия, шешім қабылдау ағашы және градиенттік бустинг әдістері көмегімен болжаудың төрт моделін пайдалана отырып, Алматы қаласындағы Кіші Алматы өзені бассейніндегі су тасқынының болуын болжауға арналған.

Түйінді сөздер: су тасқынын болжау, машиналық оқыту, кездейсоқ орман ағашы әдісі, сызықтық регрессия әдісі, шешім қабылдау ағашы әдісі және градиенттік бустинг әдісі.

Кабдрахова С.С.

Прогнозирование будущих наводнений в Алматинской области с использованием методов машинного обучения

Аннотация. Развитие современной науки и техники позволяет нам реализовать многие возможности, которые еще не доступны. Например, изучение различных природных явлений и их скрытых опасностей, а также прогнозирование могут предотвратить или усилить профилактические мероприятия. Регион, наводнения и частота его возникновения влияют на

нашу жизнь в беспрецедентной степени. Очень трудно предотвратить эти события в краткосрочной перспективе, но план предотвращения рисков может уменьшить негативные последствия аварии. Настоящее исследование посвящено оценке потенциала наводнений в бассейне реки Малая Алматинка в Алматы с использованием четырех моделей прогнозирования случайного леса, линейной регрессии, дерево решений и градиентого бустинга.

Ключевые слова: прогнозирование наводнения, машинное обучение, метод случайного леса, метод линейной регрессии, метод дерева решений и метод градиентного бустинга.

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BOUNDED SOLUTIONS OF DIFFERENTIAL SYSTEMS WITH SINGULARITIES AND THEIR APPROXIMATIONS

Abstract. *Singular boundary value problems for a linear nonhomogeneous system of ordinary differential equations on a finite interval are considered. It is supposed that improper integrals of the norm of the coefficient matrix over semiaxes are infinite*

Key words: *ordinary differential equations, singular boundary value problem, bounded solution, approximation, behavior of solutions at singular points, the parameterization method.*

Numerous application problems give rise to differential equations on an infinite interval or with singularities at an endpoint. Various problems for such equations have been studied by many authors (see [1–8] and references therein). A survey of results on singular boundary value problems for second order ordinary differential equations, as well as examples of specific physical processes leading to them, can be found in [4].

It is known that one of the main issues of the theory of singular problems is the problem of their approximation by regular boundary value problems. The resolution of this problem allows us not only to construct an approximate method for finding solutions to singular boundary value problems, but also to establish effective criteria for their well-posedness in terms of approximating regular boundary value problems.