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DZHUMABAEV PARAMETERIZATION METHOD FOR SOLVING AN INITIAL–BOUNDARY VALUE PROBLEM FOR HIGHER ORDER PARTIAL DIFFERENTIAL EQUATIONS

Abstract. *We consider an application of the Dzhumabaev parameterization method for solving initial-boundary value problems for higher order partial differential equations with two variables. These problems are reduced to nonlocal problems for system of hyperbolic equations of second order with mixed derivatives, or to the family of boundary value problems for hybrid systems consisting of first order partial differential equations, or systems of ordinary differential equations with a parameter and functional relations. A family of multipoint boundary value problems for higher order differential equations is solved by the Dzhumabaev parameterization method. The methods and results are developed to nonlocal problems for higher order partial differential equations with loading and delay arguments, nonlocal problems with integral conditions and impulse effects for higher order partial differential equations.*

Key words: *initial-boundary value problems, higher order partial differential equations, Dzhumabaev parameterization method, system of hyperbolic equations second order, nonlocal problems, unique solvability.*

Introduction.

The Dzhumabaev parameterization method was created for investigating and solving linear boundary value problems for systems of ordinary differential equations [1]. On the basis of this method, the coefficient criteria for unique solvability of linear two-point boundary value problems for systems of ordinary differential equations were established. The Dzhumabaev parameterization method and these results were developed to various classes of boundary value problems for differential equations [2-11]. Further, the Dzhumabaev parameterization method was extended to the linear two-point boundary value problems for integro-differential equations. Application of the Dzhumabaev parameterization method made it possible to establish necessary and sufficient conditions for the solvability and unique solvability of linear boundary value problems for ordinary Fredholm integro-differential equations [12-16]. Algorithms of the parameterization method for solving these problems are proposed in [17, 18]. These results are extended to nonlinear boundary value problems for ordinary Fredholm integro-differential equations and loaded differential equations [19-22]. Necessary and sufficient conditions for solvability and the unique solvability of these problems are received.

The theory of nonlocal boundary value problems for systems of second order hyperbolic equations has been developed in the work of many authors. At present, different conditions for solvability of nonlocal boundary value problems for hyperbolic equations have been received. Criteria for unique solvability of some classes of linear boundary value problems for hyperbolic equations with variable coefficients have been obtained quite recently. We are proposing a method of introducing of functional parameters for solving nonlocal boundary value problems for ayatem of hyperbolic equations of the second order. The method of introduction of functional parameters is a modification and generalization of the Dzhumabaev parameterization method to partial differential equations with two variables. By means of the method of introduction of functional parameters, the nonlocal boundary value problems for the systems of hyperbolic equations with mixed derivatives were investigated, the algorithms for finding the solutions are constructed, and the conditions for the existence of unique classical solution to the considered problem are obtained [23-25].

Using new unknown functions in [26, 27] the nonlocal boundary value problem with data on the characteristics for the systems of hyperbolic equations was reduced to the problem, which consists of a family of two-point boundary value problems for ordinary differential equations and the functional relations. It is established that the well-posed solvability of nonlocal boundary value problems with data on the characteristics for the systems of hyperbolic equations is equivalent to the well-posed solvability of a family of two-point boundary value problems for the systems of ordinary differential equations. Criteria of well-posed solvability of linear nonlocal boundary value problems for systems of hyperbolic equations with mixed derivatives in terms of initial data are obtained. These results are extended to a nonlocal boundary value problem with integral condition for the system of hyperbolic equations [28], nonlocal boundary value problems for system of loaded hyperbolic equations [29] and periodic problems for the system of hyperbolic equations with finite time delay [30].

Main results.

Currently, the problems of mathematical physics connected with the description of the wave motion of liquids of different nature are drawing great attention. This interest is caused not only by the significant applied importance of these problems, but their new theoretical and mathematical content often do not have analogues in classical mathematical physics. One of the important classes of such problems are the initial-boundary value problems for higher order partial differential equations. To date, various methods for researching and solving the initial-boundary value problems for higher order partial differential equations of hyperbolic and composite types have been developed (see bibliography in [31]). In order to investigate various boundary value problems for higher order partial differential equations along with the classical methods of mathematical physics (the Fourier method, the method of Green's functions, Poincare's metric concept) we apply the method of differential inequalities and other methods of qualitative theory of ordinary differential equations. Based on them, the conditions for solvability of considered boundary value problems are obtained, and the ways for finding their solutions are offered. However, finding the effective signs of unique solvability of initial-boundary value problems, the analog of multipoint boundary value problems for higher order partial differential equations, still remains an active problem.

It is known that higher order ordinary differential equations can be reduced to a system of first order ordinary differential equations by replacement. Using the methods of the qualitative theory of differential equations for the received system conditions of solvability can be formulated in the terms of a fundamental matrix of a differential part or the right part of system . A similar approach can be applied to higher order hyperbolic equations with two independent variables and the equations can be reduced to the system of second order hyperbolic equations with mixed derivatives by replacement. Then, using the known methods for solving boundary value problems for systems of hyperbolic equations with mixed derivatives, the conditions of solvability can be established in different terms. Mathematical modeling of many problems of physics, mechanics, chemistry, biology, etc., resulted in the necessity of research of multipoint and nonlocal boundary value problems for higher order partial differential equations of the hyperbolic type. Applying the methods of qualitative theory of differential equations directly to these problems, we can establish conditions of their solvability. Also by means of replacement, the multipoint and nonlocal boundary value problems for higher order partial differential equations of the hyperbolic type are reduced to the nonlocal boundary value problems for systems of second order hyperbolic equations.

In the present paper, the Dzhumabaev parameterization method and its modifications are the basic methods used to investigate and solve the initial-boundary value problems for higher order partial differential equations. We establish the conditions for unique solvability of initial-boundary value problems, the analog of multipoint boundary value problems for higher order partial differential equations of the hyperbolic type. Criteria for the well-posed solvability of family of multipoint boundary value problems for higher order differential equations are received. Special attention in the article is given to the initial-boundary value problems for third and fourth order partial differential equations, which often arise in the mathematical modeling of processes of the movement of stratified liquid, during investigation research of an ion-sound wave in non-magnetized plasma, and when studying the wave processes in various environments and rheological schemes of crust [32-38]. Conditions for the solvability of initial-boundary value problems, and of nonlocal boundary value problems for these equations are obtained by the method of characteristics, Riemann's method, the method of Green's functions and differential inequalities. The conditions for the unique solvability of initial-boundary value problems and analog multipoint boundary value problems for third and fourth order partial differential equations are established in the terms of solvability to nonlocal boundary value problems for systems of hyperbolic equations of the second order. Considered problems also are reduced to the family of boundary value problems for hybrid systems consisting of first order partial differential equations and ordinary differential equations with parameter and functional relations. On the basis of the Dzhumabaev parameterization method, the algorithms for finding the solution and coefficient conditions for the well-posed solvability of investigated problem are proposed.

The theory of boundary value problems for higher order partial differential equations is closely related to the theory of boundary value problems for higher order ordinary differential equations. Conditions for the existence of solutions to the boundary value problem and Vallee-Poussin problem for higher order hyperbolic equations are established, using the properties (the existence of solutions, uniqueness of solutions, the continuous dependence on initial data) of the family of corresponding homogeneous boundary value problems for higher order ordinary differential equations. Applying the method of the introduction of functional parameters to the nonlocal boundary value problems for systems of hyperbolic equations also led to the family of boundary value problems for ordinary differential equations. Using the fact that well-posed solvability of boundary value problems with data on the characteristics for systems of hyperbolic equations is equivalent to the well-posed solvability of the family of two-point boundary value problems for ordinary differential equations, made it possible to establish a criterion for well-posed solvability. In addition, the families of boundary value problems for higher order ordinary differential equations are of independent interest as non-Fredholm problems. In this regard, the project will investigate the family of multipoint boundary value problems for higher order differential equations by the Dzhumabaev parameterization method. On the basis of this method, the coefficient criteria for unique solvability of linear regular boundary value problems for the systems of ordinary differential equations were established. By means of modification of the parameterization method, the algorithms for finding the solutions are proposed, and the conditions for well-posed solvability of the family of multipoint boundary value problems for higher order differential equations in terms of the initial data are established [39, 40].

The principal difference between the Dzhumabaev parameterization method and these results from the existing analogs consists in the establishment of coefficient conditions for the existence of solutions to the specified problems and in the construction effective algorithms for finding their solutions.

Thus, the Dzhumabaev parameterization method and its modifications for finding the approximate solutions to the initial-boundary value problems, the families of multipoint boundary value problems for higher order differential equations, the boundary value problems for higher order hyperbolic equations with loading and with a delay argument, the nonlocal problems with integral conditions, with impulse effects of higher order partial differential equations will be offered, and the conditions for the existence of a solution in the terms of initial data will be established.

Conclusion

The theory of boundary value problems for higher order partial differential equations is actively developing, and it finds numerous applications in various fields of applied mathematics. Works of many authors are devoted to the research of these problems and development of methods for finding their solutions. Study of the qualitative properties of initial-boundary value problems, multipoint boundary value problems and the development of effective methods for finding their solutions are the main problems of this theory. The coefficient conditions for unique solvability of initial-boundary value problems and a family of multipoint boundary value problems for higher order partial differential equations, which will be established as well as the approximate methods which will be developed in this paper, will become a powerful contribution to the theory of boundary value problems for higher order partial differential equations.

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Жоғарғы ретті дербес туындылы дифференциалдық теңдеулер үшін бастапқы-шеттік есептерді шешуге арналған Жұмабаевтың параметрлеу әдісі

Аңдатпа: Екі айнымалысы бар жоғарғы ретті дербес туындылы дифференциалдық теңдеулер үшін бастапқы-шеттік есептерді шешуге арналған Жұмабаевтың параметрлеу әдісінің кейбір қолданыстары қарастырылады. Бұл есептер аралас туындылы гиперболалық теңдеулер жүйесі үшін бейлокал есептерге, немесе бірінші ретті дербес туындылы дифференциалдық теңдеулерден тұратын гибридті жүйелер үшін шеттік есептер әулетіне, немесе параметрлері бар жай дифференциалдық теңдеулер жүйелері мен функционалдық қатынастарға келтіріледі. Жоғарғы ретті дифференциалдық теңдеулер үшін көпнүктелі шеттік есептер әулеті Жұмабаевтың параметрлеу әдісі арқылы шешіледі. Әдістер мен алынған нәтижелер жүктемелері мен кешігулі аргументтері бар жоғарғы ретті дербес туындылы дифференциалдық теңдеулер үшін бейлокал есептерге, жоғарғы ретті дербес туындылы дифференциалдық теңдеулер үшін интегралдық шарттары мен импульстік әсерлері бар бейлокал есептерге дамытылады.

Түйінді сөздер: бастапқы-шеттік есептер, жоғарғы ретті дербес туындылы дифференциалдық теңдеулер, Жұмабаевтың параметрлеу әдісі, екінші ретті гиперболалық теңдеулер жүйесі, бірмәнді шешілімділік

Иманчиев А.Е.*, Абильдаева А.Д., Минглибаева Б.Б.

Метод параметризации Джумабаева решения начально-краевых задач для дифференциальных уравнений в частных производных высокого порядка

Аннотация. Рассматриваются некоторые применения метода параметризации Джумабаева для решения начально-краевых задач для дифференциальных уравнений в частных производных высокого порядка с двумя переменными. Эти задачи сводятся к нелокальным задачам для системы гиперболических уравнений второго порядка со смешанными производными, или к семейству краевых задач для гибридных систем, состоящих из дифференциальных уравнений с частными производными первого порядка, или к системам обыкновенных дифференциальных уравнений с параметрами и функциональным соотношениям. Семейство многоточечных краевых задач для дифференциальных уравнений высокого порядка решается методом параметризации Джумабаева. Методы и полученные результаты развиты на нелокальные задачи для дифференциальных уравнений в частных производных высокого порядка с нагружениями и запаздывающими аргументами, нелокальные задачи с интегральными условиями и импульсными воздействиями для уравнений в частных производных высокого порядка.

Ключевые слова: начально-краевые задачи, дифференциальные уравнения в частных производных высокого порядка, метод параметризации Джумабаева, система гиперболических уравнений второго порядка, однозначная разрешимость.

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