РАЗРАБОТКА ПРОГРАММНОГО ОБЕСПЕЧЕНИЯ И ИНЖЕНЕРИЯ ЗНАНИЙ

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RESEARCH OF A NONLOCAL BOUNDARY VALUE PROBLEM BY THE PARAMETERIZATION METHOD

Abstract. *A nonlocal boundary value problem with an integral condition for a system of second order partial differential equations is considered. Sufficient coefficients conditions of well-posed solvability of the problem are obtained by the parameterization method as well as an algorithm for finding a solution are offered.*

Keywords: *integral condition, nonlocal boundary value problem, Friedrichs, algorithm.*

Introduction

Among boundary value problems for partial differential equations, problems in which the conditions connect the desired solution and its derivatives at various points lying on the border or inside the considered area are of considerable interest. Boundary value problems with nonlocal conditions for a wide class of partial differential equations have been studied by many authors using various methods. Note the works [1]-[2], where you can find a detailed overview and bibliography on these problems.

Finding effective signs of the solvability of boundary value problems for some classes of partial differential equations, developing new effective approaches to the study of boundary value problems, and developing iterative methods for partial differential equations are relevant both for expanding the class of well-posed solvable boundary value problems, and for applying mathematical methods to the problems under study.

Boundary value problems for systems of hyperbolic equations with mixed derivative are investigated and solved by the method of introduction of functional parameters [3], which is a modification of the parameterization method [4] developed by Doctor of Physical and Mathematical Sciences, Professor D. S. Dzhumabaev for solving boundary value problems of ordinary differential equations.

Nonlocal problems with integral conditions arise in mathematical modelling of various physical phenomena. Nonlocal boundary value problems with integral conditions for partial differential equations began to be studied relatively recently. In [5], a nonlocal boundary value problem with integral condition for a time variable for the system of hyperbolic equations with a mixed derivative is considered.

Problem statement

The nonlocal boundary value problem for the system of partial differential equations

$$D\left[\frac{\partial}{\partial x}u\right] = A(x,t)\frac{\partial u}{\partial x} + S(x,t)u + f(x,t), \qquad u \in R^n, \tag{1}$$

$$B(x)\frac{\partial u}{\partial x}(x,0)\Big|_{x\in[0,\omega]} + C(x)\frac{\partial u}{\partial x}(x,T)\Big|_{x\in[\mathrm{T},\mathrm{T}+\omega]} + \int_0^T K(x,s)\frac{\partial u}{\partial x}(x,s)ds = d(x), \tag{2}$$

$$u(t,t)=\Psi(t), \qquad t\in[0,T] \tag{3}$$

is considered in $\overline{\Omega}=\{(x,t): t\le x\le t+\omega, 0\le t\le T\}$, $T>0$, $\omega>0$.

Here, $u(x,t)=col(u_1(x,t),u_2(x,t),...,u_n(x,t))$ is an unknown function; $D=\frac{\partial}{\partial t}+\frac{\partial}{\partial x}$; $A(x,t)$, $S(x,t)$, $K(x,t)$ are $(n\times n)$ matrices, n is vector-function $f(x,t)$, $(n\times n)$ are matrices $B(x)$, $C(x)$, n is vector-function $d(x)$ and is function $\Psi(t)$ continuous on $\overline{\Omega}$, $[0,\omega]$, $[0,T]$ accordingly.

Let $C(\overline{\Omega},R^n)$ be a space of functions $u:\overline{\Omega}\to R^n$ that are continuous on $\overline{\Omega}$, with the norm $\|u\|_0=\max_{(x,t)\in\overline{\Omega}}\|u(x,t)\|$; $\|A\|=\max_{(x,t)\in\overline{\Omega}}\|A(x,t)\|=\max_{(x,t)\in\overline{\Omega}}\max_{i=1,n}\sum_{j=1}^{n}|a_{ij}(x,t)|$, $\|d\|_1=\max_{x\in[0,\omega]}\|d(x)\|$,

$\|\Psi\|_2=\max_{t\in[0,T]}\|\Psi(t)\|$.

In the present work, we investigate questions of well-posed solvability to the wide extent of the nonlocal boundary value problem (1)-(3).

Main results

Using the ideas of [3] and [5], we introduce new unknown functions [6] $v(x,t)=\frac{\partial u}{\partial x}(x,t)$, and the problem under study is reduced to the equivalent problem for the system of first-order hyperbolic equations

$$Dv=A(x,t)v+S(x,t)u+f(x,t), \quad (x,t)\in\overline{\Omega}, \tag{4}$$

$$B(x)v(x,0)\big|_{x\in[0,\omega]}+C(x)v(x,T)\big|_{x\in[T,T+\omega]}+\int_0^T K(x,s)v(x,s)ds=d(x), \tag{5}$$

$$u(x,t)=\Psi(t)+\int_t^x v(\eta,t)d\eta, \qquad t\in[0,T]. \tag{6}$$

A pair $(v(x,t),u(x,t))$ of continuous functions on $\overline{\Omega}$ is called a solution to problem (4)-(6) to the wide extent of Friedrichs if the function $v(x,t)\in C(\overline{\Omega},R^n)$ has a continuous derivative with respect to t along characteristic and satisfies the family of ordinary differential equations, and condition (5), in which the functions $u(x,t)$ and $v(x,t)$ are linked by the functional relation (6).

Using the method of characteristic, we get in $\overline{H}=\{(\xi,\tau): 0\le\xi\le\omega, 0\le\tau\le T\}$, $T>0$, $\omega>0$:

$$\frac{\partial\tilde{v}}{\partial\tau}=\tilde{A}(\xi,\tau)\tilde{v}+\tilde{S}(\xi,\tau)\tilde{u}(\xi,\tau)+\tilde{f}(\xi,\tau), \qquad \tau\in[0,T], \tag{7}$$

$$\tilde{B}(\xi)\tilde{v}(\xi,0)+\tilde{C}(\xi)\tilde{v}(\xi,T)+\int_0^T\tilde{K}(\xi,\tau)\tilde{v}(\xi,\tau)d\tau=\tilde{d}(\xi), \qquad \xi\in[0,\omega], \tag{8}$$

$$\tilde{u}(\xi,\tau)=\Psi(\tau)+\int_\tau^{\xi+\tau}\tilde{v}(\varsigma,\tau)d\varsigma, \qquad \tau\in[0,T], \tag{9}$$

where $\tilde{v}(\xi,\tau)=v(\xi+\tau,\tau)$, $\tilde{u}(\xi,\tau)=u(\xi+\tau,\tau)$, $\tilde{A}(\xi,\tau)=A(\xi+\tau,\tau)$, $\tilde{S}(\xi,\tau)=S(\xi+\tau,\tau)$, $\tilde{K}(\xi,\tau)=K(\xi+\tau,\tau)$, $\tilde{f}(\xi,\tau)=f(\xi+\tau,\tau)$; the $(n\times n)$ matrices $\tilde{A}(\xi,\tau)$, $\tilde{S}(\xi,\tau)$, $\tilde{K}(\xi,\tau)$, n-vector-function $\tilde{f}(\xi,\tau)$ is continuous on $\overline{H}$; $(n\times n)$ are matrices $\tilde{B}(\xi)$, $\tilde{C}(\xi)$, n-vector-function $\tilde{d}(\xi)$ is continuous on $[0,\omega]$, and n-vector-function $\Psi(\tau)$ is continuously differentiable on $[0,T]$.

Let $C(\overline{H},R^n)$ be the space of continuous functions $\tilde{v}:\overline{H}\to R^n$ on $\overline{H}$ with norm $\|\tilde{v}\|_0=\max_{\xi\in[0,\omega]}\max_{\tau\in[0,T]}\|\tilde{v}(\xi,\tau)\|$.

A continuous function $\tilde{v}(\xi,\tau)$ on $\overline{H}$ is called a solution to problems (7)-(9) if the function $\tilde{v}(\xi,\tau)\in C(\overline{H},R^n)$ has a continuous derivative with respect to τ and satisfies the family of boundary value problems for the system of ordinary differential equations, and condition (8), in which the functions $\tilde{u}(\xi,\tau)$ and $\tilde{v}(\xi,\tau)$ by the functional relation (9).

A continuous function $u(x,t)=\tilde{u}(x-t,t)$ on $\overline{\Omega}$ is called a solution to the wide extent of boundary value problems for the system of partial differential equations (1) with nonlocal integral conditions (2) and (3).

For solving boundary value problems (7)-(9), we offer the following algorithm.

Step-0: in (7) accepting $\tilde{u}(\xi,\tau)=\Psi(\tau)$, and solving boundary value problems (7)-(8) we shall define initial approach $\tilde{v}^{(0)}(\xi,\tau)$. Using the $\tilde{v}(\xi,\tau)=\tilde{v}^{(0)}(\xi,\tau)$ from correlation (9) find $\tilde{u}^{(0)}(\xi,\tau)$.

Step-1: taking $\tilde{u}(\xi,\tau)=\tilde{u}^{(0)}(\xi,\tau)$ in the right-hand side of (7) and solving boundary value problems (7)-(8), we define initial approximation $\tilde{v}^{(1)}(\xi,\tau)$. Substituting in (9) the function $\tilde{v}^{(1)}(\xi,\tau)$ found, we find $\tilde{u}^{(1)}(\xi,\tau)$.

And so on.

On step k : continuing this process we get $\left(\tilde{v}^{(k)}(\xi,\tau),\tilde{u}^{(k)}(\xi,\tau)\right)$.

On each step of the offered algorithm we use the parameterization method [4].For fixed $\tilde{u}(\xi,\tau)$, $\xi\in[0,\omega]$, problems (7)-(8) become the problems for equations

$$\frac{\partial\tilde{v}}{\partial\tau}=\tilde{A}(\xi,\tau)\tilde{v}+\tilde{G}(\xi,\tau),\qquad \tau\in[0,T] \tag{10}$$

with condition (8).

A continuous function $\tilde{v}:\overline{H}\to R^n$ that has a continuous derivative with respect to τ on $\overline{H}$ is called a solution of the family of two point boundary value problems (10) and (8) if it satisfies system (10) and condition (8) for all $(\xi,\tau)\in\overline{H}$ and $\xi\in[0,\omega]$, respectively.

Definition 1. A family of two point boundary value problems (10) and (8) is said to be well-posed and solvable if it has a unique solution $\tilde{v}(\xi,\tau)\in C(\overline{H},R^n)$ for any functions $\tilde{G}(\xi,\tau)$ and $\tilde{d}(\xi)$ and this solution satisfies the estimate

$$\max_{\tau\in[0,T]}\|\tilde{v}(\xi,\tau)\|\le\tilde{K}(\xi)\max\left(\max_{\tau\in[0,T]}\|\tilde{G}(\xi,\tau)\|,\|\tilde{d}\|_1\right),$$

where $\tilde{K}(\xi)$ is a continuous function on $[0,\omega]$ independent of $\tilde{G}(\xi,\tau)$ and $\tilde{d}(\xi)$.

Definition 2. The boundary value problems (1)-(3) are said to be well-posed solvable if they have a unique solution $u^*(x,t)\in C(\overline{\Omega},R^n)$ for any functions $f(x,t)$, $d(x)$ and $\Psi(t)$ and this solution satisfies the estimate

$$\max\left(\|u\|_0,\left\|\frac{\partial u}{\partial x}\right\|_0\right)\le K\max\left(\|f\|_0,\|d\|_1,\|\Psi\|_2\right),$$

where $K=const$ independent of $f(x,t)$, $d(x)$, $\Psi(t)$.

To solve families of two-point boundary value problems for ordinary differential equations, the method of parameterization [4] is used.

Sufficient conditions are obtained for the unique and well-posed solvability of the problem in the terms of invertibility of the matrix, and boundary condition.

Since problems (7)-(9) are equivalent to problems (4)-(6), as well as boundary value problems (4)-(6) being equivalent to (1)-(3), the nonlocal boundary value problem with integral condition for the system of partial differential equations of the second order (1)-(3) has the unique solution $u^*(x,t)\in C(\overline{\Omega},R^n)$.

Theorem. *Let boundary value problems (10) and (8) be well-posed. Then the sequence* $\left(\tilde{v}^{(k)}(\xi,\tau),\tilde{u}^{(k)}(\xi,\tau)\right)$ *converges to the unique solution of problems (7)-(9), and nonlocal boundary value problems (1)-(3) are solvable in the wide extent.*

Conclusion

When investigating and solving a nonlocal boundary value problem for a system of partial differential equations, the parameterization method is used, which allows us to establish the well-posed solvability of the problem along with unique solvability. The coefficient conditions for well-posed solvability of a nonlocal boundary value problem for a system of equations are established. Sufficient conditions for the well-posed solvability of a boundary value problem with a nonlocal condition are established in terms of a matrix formed through the right side of the equation system and the boundary condition.

If a solution built to the wide extent, is continuously differentiable with respect to x and t, i.e. $u(x,t)$ has continuous partial derivatives $\frac{\partial u}{\partial t}$, $\frac{\partial u}{\partial x}$, $D\left[\frac{\partial}{\partial x}u\right]$ and satisfies equation (1) for all $(x,t)\in\overline{\Omega}$ and conditions (2)-(3), then it is a classical solution of nonlocal boundary value problems (1)-(3).

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Абдикаликова Г.А.

Исследование нелокальной краевой задачи методом параметризации

Аннотация. Рассматривается нелокальная краевая задача с интегральным условием для системы уравнений в частных производных второго порядка.

Методом параметризации получены коэффициентные достаточные условия корректной разрешимости задачи, а также предложен алгоритм нахождения решения.

Ключевые слова: интегральное условие, нелокальная краевая задача, Фридрихс, алгоритм.

Абдикаликова Г.А.

Параметрлеу әдісі арқылы локалды емес шеттік есепті зерттеу

Аңдатпа: Екінші ретті дербес туындылы теңдеулер жүйесі үшін интегралдық шартты локалды емес шеттік есеп қарастырылған. Параметрлеу әдісі арқылы есепің корректілі шешілімділігінің жеткілікті шарттары алынған және шешімді табу алгоритмі ұсынылған.

Түйінді сөздер: интегралдық шарт, локалды емес шеттік есеп, Фридрихс, алгоритм.

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DZHUMABAEV PARAMETERIZATION METHOD FOR SOLVING AN INITIAL–BOUNDARY VALUE PROBLEM FOR HIGHER ORDER PARTIAL DIFFERENTIAL EQUATIONS

Abstract. *We consider an application of the Dzhumabaev parameterization method for solving initial-boundary value problems for higher order partial differential equations with two variables. These problems are reduced to nonlocal problems for system of hyperbolic equations of second order with mixed derivatives, or to the family of boundary value problems for hybrid systems consisting of first order partial differential equations, or systems of ordinary differential equations with a parameter and functional relations. A family of multipoint boundary value problems for higher order differential equations is solved by the Dzhumabaev parameterization method. The methods and results are developed to nonlocal problems for higher order partial differential equations with loading and delay arguments, nonlocal problems with integral conditions and impulse effects for higher order partial differential equations.*

Key words: *initial-boundary value problems, higher order partial differential equations, Dzhumabaev parameterization method, system of hyperbolic equations second order, nonlocal problems, unique solvability.*

Introduction.

The Dzhumabaev parameterization method was created for investigating and solving linear boundary value problems for systems of ordinary differential equations [1]. On the basis of this method, the coefficient criteria for unique solvability of linear two-point boundary value problems for systems of ordinary differential equations were established. The Dzhumabaev parameterization method and these results were developed to various classes of boundary value problems for differential equations [2-11]. Further, the Dzhumabaev parameterization method was extended to the linear two-point boundary value problems for integro-differential equations. Application of the Dzhumabaev parameterization method made it possible to establish necessary and sufficient conditions for the solvability and unique solvability of linear boundary value problems for ordinary Fredholm integro-differential equations [12-16]. Algorithms of the parameterization method for solving these problems are proposed in [17, 18]. These results are extended to nonlinear boundary value problems for ordinary Fredholm integro-differential equations and loaded differential equations [19-22]. Necessary and sufficient conditions for solvability and the unique solvability of these problems are received.

The theory of nonlocal boundary value problems for systems of second order hyperbolic equations has been developed in the work of many authors. At present, different conditions for solvabil-